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Exceptional supersymmetric standard models with non-abelian discrete family symmetry

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Abstract: We introduce a non-Abelian discrete Δ_{27} family symmetry into the recently proposed classes of Exceptional Supersymmetric Standard Model (E_6SSM) based on a broken E_6 Grand Unified Theory (GUT) in order to solve the flavour problem in these models and in particular to account for tri-bimaximal neutrino mixing. We consider both the minimal version of the model (the ME$_6$SSM) with gauge coupling unification at the string scale and the E$_6$SSM broken via the Pati-Salam chain with gauge coupling unification at the conventional GUT scale. In both models there are low energy exotic colour triplets with couplings suppressed by the symmetries of the model, including the family symmetry. This leads to suppressed proton decay and long lived TeV mass colour triplet states with striking signatures at the LHC.

Keywords: Neutrino Physics, Beyond Standard Model, Quark Masses and SM Parameters, GUT.

Contents

1. Introduction

For more than thirty years the Standard Model has provided the most accurate theoretical description of particle physics and, at present, there is little direct experimental evidence to suggest that this model should be replaced with a new theory. But, despite being experimentally sound, it is widely acknowledged that the model is theoretically unsatisfactory in a number of areas [1]. For instance, a large amount of fine-tuning is required to stabilize the Higgs mass at the electroweak scale when the cut-off is taken to be a high energy scale such as the Planck scale. There is also a lack of explanation for the observed structure of the quark and lepton masses and CKM matrix elements, no explanation for the observed small neutrino masses and bi-large mixing angles, and, perhaps most importantly, the Standard Model is incompatible with General Relativity, our most accurate theory of gravity.

The most popular solution to the fine-tuning problem of the Higgs mass is to treat the Standard Model as a low energy effective field theory approximation to the Minimal Supersymmetric Standard Model (MSSM) [2]. This can also potentially explain what dark matter consists of and implies that there is unification of the Standard Model forces within the framework of a Supersymmetric Grand Unified Theory (SUSY GUT) at around 10^{16} GeV. However, the MSSM does not entirely free the Standard Model of problems with the Higgs mass since it introduces the μ-problem [3] (the unexplained origin of the SUSY Higgs-Higgsino mass parameter with a TeV scale value) and, because no superpartners have been experimentally observed so far, a small fine-tuning problem. Another related problem of SUSY GUTs is the question of how to split the colour triplet Higgs apart from their Higgs doublet partners, giving GUT scale masses to the former and weak scale masses to the latter, while also satisfying the colour-triplet induced proton decay experimental bounds.

An elegant solution to the μ-problem is to extend the particle content of the MSSM by introducing a Standard Model singlet S that couples to the Higgs doublets such that its dynamically generated vacuum expectation value (VEV) provides an effective TeV scale μ-term that is related to the breaking of Supersymmetry [4]. In such theories there is also some advantage to be gained by having an additional low energy Abelian gauge group factor $\mathrm{U}(1)'$. Without a $\mathrm{U}(1)'$ gauge group a Goldstone boson would be created by the singlet's VEV since the extended MSSM superpotential has an associated global $\mathrm{U}(1)$ symmetry [5]. Alternative ways to resolve the would-be Goldstone boson problem certainly exist, namely the global $\mathrm{U}(1)$ symmetry can be explicitly broken in some way e.g. by adding an S^3 term to the superpotential, as in the Next-to(N)MSSM [6]. However, such approaches are always accompanied by additional problems, for example, the S^3 term introduces dangerous domain walls when a Z_3 discrete symmetry associated with the NMSSM superpotential is broken. By contrast the $\mathrm{U}(1)'$ gauge group eats the Goldstone boson, resulting in an observable massive Z'.

Standard Model (SM) singlets and $\mathrm{U}(1)'$ gauge groups that can resolve the μ-problem of the MSSM as discussed above turn out to be naturally contained within SUSY GUTs based on an E_6 gauge group. In this paper we concentrate on $\mathrm{U}(1)'$ subgroups of E_6 for which the right-handed neutrinos are singlets so that a conventional see-saw mechanism can be used. E_6 models that contain a $\mathrm{U}(1)'$ for which the right-handed neutrinos are singlets have been collectively called Exceptional Supersymmetric Standard Models. Here we study two such models: the ME$_6$SSM (Minimal Exceptional Supersymmetric Standard Model) [8] and the usual E$_6$SSM [7]. Note that in both versions of the model the TeV scale spectrum involves the matter content of three complete 27 supermultiplets of E_6 in order to cancel all the gauge anomalies family by family. This means that, compared to the MSSM, there are an additional three families of extra states with the quantum numbers of three $5 + \overline{5}$ representations of SU(5) at the TeV scale. These states will obviously ameliorate the little fine-tuning of the MSSM, since they increase the lightest Higgs mass considerably [7].

In the usual E$_6$SSM [7] the $\mathrm{U}(1)'$ gauge group, called $\mathrm{U}(1)_N$, is a combination of the $\mathrm{U}(1)_\psi$ and $\mathrm{U}(1)_\chi$ Abelian subgroups of E_6 defined by $E_6 \supset \mathrm{SO}(10) \times \mathrm{U}(1)_\psi$ and $\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)_\psi$. The combination of the groups is chosen so that the right-handed neutrinos do not transform under $\mathrm{U}(1)_N$. To cancel gauge anomalies for this group, three copies of a 27 supermultiplet of E_6 survive to low energies in the model. On top of this, two additional electroweak doublets (which have opposite $\mathrm{U}(1)_N$ charges) are added at the TeV scale so that unification of the Standard Model gauge coupling constants occurs at the GUT scale. Since the colour triplets are light in the E$_6$SSM, the proton decay operators must either be forbidden or highly suppressed. The former option is achieved using an exact Z_2^B or Z_2^L symmetry under which the colour triplets are leptoquarks or diquarks [7]. Such symmetries do not commute with the E_6 (or its SU(5) or Pati-Salam subgroups) and so the model is written in terms of its Standard Model representation.[1]

The ME$_6$SSM is a 'minimal' version of the E_6SSM. This refers to the fact that the ME$_6$SSM does not contain the two additional electroweak doublets required for unification of the gauge coupling constants at the GUT scale, which can reintroduce a μ'-problem analogous to the original μ-problem [8]. Gauge coupling unification is instead predicted to occur close to the Planck scale in the ME$_6$SSM using an intermediate Pati-Salam symmetry that is broken to the Standard Model at the conventional GUT scale. The $\mathrm{U}(1)'$ group of the ME$_6$SSM, denoted by $\mathrm{U}(1)_X$, is not the same as the $\mathrm{U}(1)_N$ but still allows for a conventional see-saw mechanism since the right-handed neutrinos remain neutral under it. Unlike the E$_6$SSM, the proton decay operators are highly suppressed rather than forbidden. Since the E_6 symmetry is predicted to reside at the scale at which quantum gravity effects should dominate, the model is formulated in terms of the intermediate Pati-Salam and $\mathrm{U}(1)_\psi$ gauge groups.

In this paper we shall consider the E$_6$SSM as being broken via the Pati-Salam chain as in the ME$_6$SSM. In this case the only difference between the two models is that the E$_6$SSM involves an additional two low energy electroweak doublets, leading to unification at the GUT scale. Since, in the case of such an E$_6$SSM, the Pati-Salam gauge group does not survive for very long before it is broken, the phenomenology of the two alternative breaking chains for the E$_6$SSM (Pati-Salam or SU(5)) is very similar, differing only by the discussion of triplet decay and proton decay. For the E$_6$SSM as broken via the Pati-Salam chain, the triplet decay and proton decay discussion is the same as in the ME$_6$SSM. When the $\mathrm{SU}(2)_R$ and $\mathrm{SU}(4)_{\mathrm{PS}}$ Pati-Salam gauge coupling constants are equal to each other (as in the Pati-Salam version of the E$_6$SSM) then the charge of the $\mathrm{U}(1)_X$ group becomes equivalent to the charge of the $\mathrm{U}(1)_N$ group (see [8] for a detailed explanation of this).

Despite their obvious attractions, as outlined above, none of the E$_6$SSM models so far proposed addresses the flavour problem, i.e. provides an explanation for the structure of quark and lepton masses and mixing angles. In the past decade, the flavour problem has been enriched by the discovery of neutrino mass and mixing, leading to an explosion of interest in this area [9]. A common approach is to suppose that the quarks and leptons are described by some family symmetry which is spontaneously broken at a high energy scale [10]. In particular, the approximately tri-bimaximal nature of lepton mixing provides a renewed motivation for the idea that the Yukawa couplings are controlled by a spontaneously broken non-Abelian family symmetry which spans all three families, for example SU(3) [11, 12], SO(3) [13], or one of their discrete subgroups [15, 14]. In such models tri-bimaximal neutrino mixing arises from a combination of vacuum alignment and (constrained) sequential dominance [16]. Furthermore, such family symmetries provide a solution to the SUSY flavour and CP problems [17].

The purpose of the present paper is to extend the above classes of E$_6$SSM models to include a discrete non-Abelian family symmetry as a step towards solving the flavour problem in these models. In particular, we shall use the Δ_{27} family symmetry introduced

[1]Alternatively the theory can be written in terms of several split 27 multiplets so that the Z_2^B or Z_2^L symmetries do commute with E_6 [7].

in [14] (Δ_{27} is a discrete non-Abelian subgroup of SU(3)). This is convenient since the Δ_{27} family symmetry model in [14] and the ME$_6$SSM in [8] are both based on a high-energy Pati-Salam symmetry. Following this approach we can also construct models based on the E$_6$SSM with a Δ_{27} family symmetry which are broken through the Pati-Salam chain, as discussed in the previous paragraph. The detailed strategy we shall pursue is as follows. We will introduce the Δ_{27} family symmetry from [14] to the intermediate Pati-Salam symmetry of the ME$_6$SSM or E$_6$SSM to build a model based on a $\Delta_{27} \times G_{4221}$ gauge group where $G_{4221} \equiv \times \mathrm{SU}(4)_{\mathrm{PS}} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_\psi$. The resulting model can explain the observed mixing angles and mass spectrum of the quarks and leptons, provide a tri-bimaximal mixing for the neutrinos, solve the μ-problem and small fine-tuning problem, and does not involve doublet-triplet splitting. A novel feature of the ME$_6$SSM and the Pati-Salam formulation of the E$_6$SSM is that proton decay is suppressed in a new way by the assumed Δ_{27} family symmetry and an E_6 singlet. We also show how the μ'-problem can be solved in the E$_6$SSM using the E_6 singlet that gets an intermediate VEV and suppresses proton decay.

The layout of the remainder of the paper is as follows. In section 2 we propose a model based on the Pati-Salam gauge groups of the ME$_6$SSM or E$_6$SSM and the Δ_{27} family symmetry model. In section 3 we discuss gauge coupling unification at the GUT scale in the E$_6$SSM or at the string scale in the ME$_6$SSM. Section 4 summarizes our results.

2. ME$_6$SSM or E$_6$SSM with a Δ_{27} family symmetry

In this section we introduce a Δ_{27} family symmetry into the ME$_6$SSM or E$_6$SSM broken via the Pati-Salam chain. The E$_6$SSM model with Δ_{27} family symmetry has gauge coupling unification at the GUT scale, rather than the string scale. The resulting models are very powerful since they can address the observed mixing angles and mass spectrum of the quarks and leptons, including the tri-bimaximal mixing for the neutrinos, the μ-problem and the little fine-tuning problem of the MSSM. We also show how the model solves the problem of rapid proton decay (and colour triplet decay) without introducing doublet-triplet splitting.

In the ME$_6$SSM or E$_6$SSM the Standard Model quarks and leptons come from three copies of the fundamental E_6 multiplet of dimension 27. Each 27 multiplet breaks into the following Pati-Salam representations: $27 \to F + F^c + h + \mathcal{D} + S$ where F, F^c contain one generation of the leptons and quarks (and a charge conjugated neutrino), h can contain the MSSM Higgs bosons, $\mathcal{D}$ is often called a colour triplet Higgs since it transforms as a colour triplet, and S is a singlet of the Standard Model. The explicit Pati-Salam representations of these states are listed in table 1. Following the ME$_6$SSM and E$_6$SSM we take the third copy of the 27 multiplets to contain the MSSM Higgs bosons, which we denote by h_3. In the Δ_{27} family symmetry model in [14] the three generations of the leptons and quarks F, F^c transform as triplets under the Δ_{27} group, and the MSSM Higgs bosons h_3 transform as a singlet. The rest of the ME$_6$SSM and E$_6$SSM states from the three 27 multiplets are not considered in the family symmetry model. In sections 2.2 to 2.5 we explain the chosen Δ_{27} assignments for these ME$_6$SSM or E$_6$SSM states, which are summarized by table 1. The only distinction between the ME$_6$SSM and E$_6$SSM is that the latter involves an additional pair of electroweak doublets $h', \overline{h}'$ in order to achieve unification at the GUT scale.

We now briefly explain the approach to understanding Yukawa hierarchies and neutrino tri-bimaximal (TB) mixing via broken family symmetry, vacuum alignment and constrained sequential dominance (CSD) (for more details see [11, 12, 14, 16]). The family symmetry is broken by extra Higgs scalars called flavons, often denoted by ϕ and $\overline{\phi}$. The flavons typically couple to the SM matter fermions via heavy messenger fields giving rise (upon integrating out the messenger sector) to effective Yukawa operators proportional to powers of the flavon fields suppressed by powers of the messenger mass M. The effective Yukawa couplings are then expressed in terms of ratios of flavon vacuum expectation values (VEVs) $\langle\overline{\phi}\rangle$ to these messenger mass scales M, which defines a set of expansion parameters $\varepsilon \equiv \langle\overline{\phi}\rangle/M$. If the neutrino masses are assumed to originate from the seesaw mechanism, the TB mixing pattern receives a natural explanation by means of the so-called constrained sequential dominance mechanism. The basic idea is that only one right-handed (RH) neutrino contributes dominantly to the atmospheric neutrino mass and thus the atmospheric mixing angle corresponds to a simple ratio of Yukawa couplings of just the dominant RH neutrino. One of the subdominant RH neutrinos is then assumed to govern the solar neutrino mass, in which case the solar mixing angle corresponds to another simple ratio of Yukawa couplings associated to this RH state. The TB mixing pattern can then be implemented by means of simple constraints on the Yukawa couplings. Since these emerge from flavon VEVs, CSD is then achieved from a proper vacuum alignment of flavons in the family space, for example $|\langle\overline{\phi}_3\rangle| \approx (0,0,1)$, $|\langle\overline{\phi}_{23}\rangle| \approx (0,1,1)$, $|\langle\overline{\phi}_{123}\rangle| \approx (1,1,1)$, up to phases.

The model is defined in table 1. In addition to the Pati-Salam, Δ_{27} and $\mathrm{U}(1)_\psi$ symmetries, extra discrete and Abelian symmetries must also be applied to constrain the model into a realistic theory. The model that we formulate here is most simply constrained using the combined symmetries $\mathrm{U}(1)_R \times \mathrm{U}(1) \times Z_2 \times Z_2^H$, where $\mathrm{U}(1)_R$ is an R-symmetry that contains the R-parity of the MSSM as a subgroup. The $\mathrm{U}(1) \times Z_2$ symmetries are adapted from [14] and the Z_2^H from [8]. In the ME$_6$SSM the E_6 symmetry is assumed to be broken to its Pati-Salam and $\mathrm{U}(1)_\psi$ groups near the string scale M_S.[2] This intermediate Pati-Salam with $\mathrm{U}(1)_\psi$ is then expected to be broken near the conventional GUT scale to the Standard Model with a $\mathrm{U}(1)'$ gauge group called $\mathrm{U}(1)_X$. In the Δ_{27} family symmetry approach one expects the $\mathrm{SU}(4)_{\mathrm{PS}}$ and $\mathrm{SU}(2)_R$ groups of the Pati-Salam symmetry to be broken by different mechanisms rather than the same one as in the ME$_6$SSM and, in section 3.2, we show that we expect the $\mathrm{SU}(4)_{\mathrm{PS}}$ and $\mathrm{SU}(2)_R$ groups to be broken at two different scales in the ME$_6$SSM with Δ_{27} family symmetry model, with $\mathrm{SU}(4)_{\mathrm{PS}}$ broken at the conventional GUT scale M_{GUT} by H_R VEVs, and $\mathrm{SU}(2)_R$ broken at the compactification scale M_C, where we assume $M_C > M_{\mathrm{GUT}}$. For the E$_6$SSM with Δ_{27} family symmetry model we show in section 3.1 that the $\mathrm{SU}(2)_R$ and $\mathrm{SU}(4)_{\mathrm{PS}}$ groups must both be broken at the GUT scale so that, in this case, $M_C = M_{\mathrm{GUT}}$.

[2]Note that we expect the E_6 symmetry to be broken at the String scale M_S rather than the Planck scale since extra states from the Δ_{27} model lower the scale of unification of the ME$_6$SSM somewhat (see section 3.2).

Field	Δ_{27}	$SU(4)_{PS} \times SU(2)_L \times SU(2)_R \times U(1)_\psi$	$U(1)_R$	$U(1)$	Z_2	Z_2^H
F	3	$(4,2,1)_{\frac{1}{2}}$	1	0	+	-
F^c	3	$(\overline{4},1,\overline{2})_{\frac{1}{2}}$	1	0	+	-
h_3 ; $h_{1,2}$	1	$(1,2,2)_{-1}$	0	0	+	+ ; -
$\mathcal{D}_{1,2,3}$	1	$(6,1,1)_{-1}$	0	0	+	-
S_3 ; $S_{1,2}$	1	$(1,1,1)_2$	2	0	+	+ ; -
$16_H = \overline{H}_R,\ H_L$	3	$(\overline{4},1,\overline{2})_{\frac{1}{2}},\ (4,2,1)_{\frac{1}{2}}$	0	0	+	+
$\overline{16}_H = H_R,\ \overline{H}_L$	$\overline{3}$	$(4,1,2)_{-\frac{1}{2}},\ (\overline{4},\overline{2},1)_{-\frac{1}{2}}$	0	0	+	+
M	1	$(1,1,1)_0$	2	0	+	+
Σ	1	$(1,1,1)_0$	0	5	-	-
H_{45}	1	$(15,1,3)_0$	0	2	+	+
ϕ_{123}	3	$(1,1,1)_0$	0	-1	+	+
ϕ_3	3	$(1,1,1)_0$	0	3	+	+
ϕ_1	3	$(1,1,1)_0$	0	-4	-	+
$\overline{\phi}_3$	$\overline{3}$	$(1,1,2\times 2)_0$	0	0	-	+
$\overline{\phi}_{23}$	$\overline{3}$	$(1,1,1)_0$	0	-1	-	+
$\overline{\phi}_{123}$	$\overline{3}$	$(1,1,1)_0$	0	1	-	+
h'; $\overline{h}'$	1	$(1,2,1)_x$, $(1,2,1)_{-x}$	1	-5	+	+

Table 1: This table lists all the particles (excluding the messengers) contained in the ME$_6$SSM and E$_6$SSM with a Δ_{27} family symmetry model where the E_6 symmetry is broken via the Pati-Salam chain. The Δ_{27} and G_{4221} representations are given for each particle, as well as the assignments for the additional constraining symmetries $U(1)_R \times U(1) \times Z_2 \times Z_2^H$. The $F, F^c, h_3, h_{1,2}, D_{1,2,3}, S_3$ and $S_{1,2}$ particles are expected to come from three copies of a 27 multiplet of a broken E_6 symmetry, the $16_H + \overline{16}_H$ are considered to be remnants of $27_H + \overline{27}_H$ E_6 states, the H_{45} is expected to come from a 650 multiplet of E_6 or as a composite of additional $27 + \overline{27}$ states, and, with the exception of $\overline{\phi}_3$, the flavons are singlets of E_6. The three copies of the 27 are the same as those in the ME$_6$SSM and E$_6$SSM. The flavons, H_{45} and H_R are the same as the equivalent particles in [14], and the $\overline{H}_R$, M and Σ particles are similar to the equivalent states in the ME$_6$SSM. In the E$_6$SSM family symmetry model there are also two additional electroweak doublets h' and $\overline{h}'$ which cause the gauge coupling constants to unify at the GUT scale and have $U(1)_\psi$ charges of $\pm x$ where x is some real number. These particles are not in the ME$_6$SSM with a Δ_{27} family symmetry model.

In the next subsection (2.1) we briefly explain how the Δ_{27} family symmetry from [14] when applied to the ME$_6$SSM or E$_6$SSM (broken via the Pati-Salam group) can explain the quark and lepton masses and mixing angles using the Yukawa interactions generated by the symmetry.

2.1 Yukawa Interactions

In the ME$_6$SSM and E$_6$SSM models considered here the F and F^c transform as Δ_{27} triplets and h_3 transforms as a singlet. This forbids the superpotential term $Y_{ij} F^i F^{cj} h_3$, where $i, j = 1 \ldots 3$ and Y_{ij} are theoretically undetermined Yukawa coefficients. Instead higher order terms are allowed that effectively generate the Standard Model Yukawa interactions but with the desired Yukawa coefficients dynamically generated to give the observed CKM matrix and quark and lepton masses. This is achieved by introducing new particles to the theory that couple to the fermions and quarks via their Δ_{27} components and break the family symmetry to nothing. These new particles are called flavons and are singlets of the Standard Model gauge group. Six such particles are required and their $G_{4221} \equiv \mathrm{SU}(4)_{\mathrm{PS}} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_\psi$ and Δ_{27} representations, as well as their $\mathrm{U}(1)_R \times \mathrm{U}(1) \times Z_2 \times Z_2^H$ charges, are given in table 1. These symmetry assignments are simply borrowed from [14].

The leading Yukawa terms allowed by the symmetries are [14]:

$$\frac{1}{M_R^2} F^i F^{cj} h_3 \overline{\phi}_{3i} \overline{\phi}_{3j} \tag{2.1}$$

$$\frac{1}{M_R^3} F^i F^{cj} h_3 H_{45} \overline{\phi}_{23i} \overline{\phi}_{23j} \tag{2.2}$$

$$\frac{1}{M_R^2} F^i F^{cj} h_3 (\overline{\phi}_{123i} \overline{\phi}_{23j} + \overline{\phi}_{123j} \overline{\phi}_{23i}) \tag{2.3}$$

$$\frac{1}{M_R^5} F^i F^{cj} h_3 H_{45} (\overline{\phi}_{3i} \overline{\phi}_{123j} + \overline{\phi}_{3j} \overline{\phi}_{123i})(\overline{\phi}_{123k} \phi_1^k) \tag{2.4}$$

$$\frac{1}{M_R^6} F^i F^{cj} h_3 \overline{\phi}_{123i} \overline{\phi}_{123j} (\overline{\phi}_{3k} \phi_{123}^k)(\overline{\phi}_{3l} \phi_{123}^l) \tag{2.5}$$

where the Latin indices refer to the Δ_{27} symmetry, and M_R is the mass of right-handed messengers, which is explained below. The H_{45} in eq. (2.2) and eq. (2.4) is a Δ_{27} singlet that transforms as $(15, 1, 3)_0$ under the G_{4221} symmetry. This particle gets a VEV in the hypercharge direction generating the Georgi-Jarlskog factor for eq. (2.2) [18, 19]. This sets $m_\mu \sim 3m_s$ at the family symmetry breaking scale, which, after radiative corrections from the Grand Unified scale in an MSSM inspired GUT, agrees well with experimental data. Since right-handed neutrinos have zero hypercharge, the H_{45} also suppresses the neutrino mass matrix. This is necessary for tri-bimaximal mixing to come from the Δ_{27} family symmetry [11].

The high order superpotential terms given by eq. (2.1)–(2.5) are assumed to come from renormalizable, high-energy interactions involving heavy vector-like particles that transform in the same way as the quark and lepton fields under the G_{4221} symmetry. Such particles, called messengers, are integrated out of the high energy theory to generate the above suppressed superpotential terms. To distinguish the Yukawa matrices for the up and down quarks we require that the $\mathrm{SU}(2)_R$ messengers dominate over the $\mathrm{SU}(2)_L$ messengers and, for the correct up and down Yukawa matrices, we require that the up and down right-handed messengers have mass M_u and M_d related by $M_u \sim \frac{1}{3} M_d$ [12, 11]. This can be achieved within the framework of Wilson-line breaking of $\mathrm{SU}(2)_R$ at some compactification scale [12]. We use M_R to denote the right-handed messenger scale, which could be M_u or M_d depending on the interactions involved.

The flavons $\overline{\phi}_3 + \phi_3$, $\overline{\phi}_{23} + \phi_1$ and $\overline{\phi}_{123} + \phi_{123}$ get VEVs of order $\epsilon_3 M_d$, $\epsilon_d M_d$ and $\epsilon_d^2 M_d$ respectively. The Δ_{27} components that get VEVs are given by the flavons' subscripts. Putting these VEVs into eq. (2.1)–(2.5) generates the following leading order up and down quark Yukawa matrices [11]:

$$Y_u \propto \begin{pmatrix} 0 & \epsilon_u^2\epsilon_d & -\epsilon_u^2\epsilon_d \\ \epsilon_u^2\epsilon_d & -2\epsilon_u^2\frac{\epsilon_u}{\epsilon_d} & 2\epsilon_u^2\frac{\epsilon_u}{\epsilon_d} \\ -\epsilon_u^2\epsilon_d & 2\epsilon_u^2\frac{\epsilon_u}{\epsilon_d} & \epsilon_3^2 \end{pmatrix} \qquad Y_d \propto \begin{pmatrix} 0 & \epsilon_d^3 & -\epsilon_d^3 \\ \epsilon_d^3 & \epsilon_d^2 & -\epsilon_d^2 \\ -\epsilon_d^3 & -\epsilon_d^2 & \epsilon_3^2 \end{pmatrix}$$

If $\epsilon_3 \sim 0.5 - 1.0$, $\epsilon_u \sim 0.05$, $\epsilon_d \sim 0.13$, then, after radiative corrections from a high energy scale, the above matrices (and corresponding lepton matrices) are able to generate quark and lepton masses and CKM values that are in good agreement with the observed values [20].

It should be noted that in both the ME$_6$SSM and E$_6$SSM with Δ_{27} family symmetry models the renormalization group equations (RGEs) will be different from those in the MSSM since there are three copies of a supersymmetric E_6 27 multiplet below the conventional GUT scale (and two additional electroweak doublets in the E$_6$SSM model) rather than just the MSSM particle spectrum. This is illustrated by figure 1 for the ME$_6$SSM with Δ_{27} family symmetry model. The Yukawa terms in the Δ_{27} model [14] were assumed to be formulated at the GUT scale and, after running the assumed MSSM from the GUT scale to the electroweak scale, the results agree with the observed quark and lepton mixing angles and masses. In the ME$_6$SSM and E$_6$SSM with Δ_{27} family symmetry models the running effects will clearly be different, but we do not expect the main features of the low energy spectrum to be qualitatively very different.

2.2 Majorana Interactions

In the ME$_6$SSM and E$_6$SSM models considered here the Pati-Salam symmetry is broken to the Standard Model by particles that transform as $(4,1,2)_{-\frac{1}{2}} + (\overline{4},1,\overline{2})_{\frac{1}{2}}$ under G_{4221}. The $(4,1,2)_{-\frac{1}{2}}$ particle, denoted by H_R, once it develops its GUT scale VEV, gives mass to the right-handed neutrinos using Planck suppressed operators $\frac{1}{M_p}\lambda_{ij}F^{ci}F^{cj}H_RH_R$. This non-renormalizable term, together with the Yukawa interaction involving the neutrinos, can explain the small mass scale of the neutrinos but not the observed hierarchical structure of neutrino masses and large mixing angles without setting the couplings λ_{ij} by hand. In the Δ_{27} family symmetry model the particles that give mass to the right-handed neutrinos also transform as $(4,1,2)$ under the Pati-Salam gauge group but are taken to transform as anti-triplets under Δ_{27}. With this Δ_{27} assignment, the particles can dynamically generate the observed hierarchical structure of neutrino masses and a tri-bimaximal mixing. Following the Δ_{27} family symmetry model, we therefore take the H_R particle to transform as an anti-triplet of Δ_{27}. The Majorana interactions are then given by [14]:

$$\frac{1}{M_R}F^{ci}F^{cj}H_{Ri}H_{Rj} \tag{2.6}$$

$$\frac{1}{M_R^5}F^{ci}F^{cj}\overline{\phi}_{23i}\overline{\phi}_{23j}H_{Rk}H_{Rl}\phi_{123}^k\phi_3^l \tag{2.7}$$

$$\frac{1}{M_R^5}F^{ci}F^{cj}\overline{\phi}_{123i}\overline{\phi}_{123j}H_{Rk}H_{Rl}\phi_{123}^k\phi_{123}^l \tag{2.8}$$

Together with the neutrino Yukawa matrix generated by eq. (2.1)–(2.5), the above interactions produce a $U_{\rm PMNS}$ matrix with tri-bimaximal mixing and a hierarchical structure of

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neutrino masses in agreement with the observed values [21]. How this happens is discussed in [14] and references therein, and the details of this are identical for the present model.

2.3 The μ-term and colour triplet Higgs Mass

Taking S_3 to transform as a singlet under Δ_{27} allows the superpotential term $S_3h_3h_3$. This term is also allowed in the ME$_6$SSM and E$_6$SSM. If S_3 obtains a vacuum expectation value at the TeV scale, $S_3h_3h_3$ will become an effective μ-term of the MSSM with the desired value of μ for electroweak symmetry breaking. The S_3 VEV is expected to depend on the breaking of SUSY [3], thus resolving the μ-problem of the MSSM.

In addition to solving the μ-problem of the MSSM, this model will also resolve the little fine-tuning problem of the MSSM. This is because there are extra particles below the conventional GUT scale of 10^{16} GeV that are not contained in the MSSM. These extra particles are from the three copies of the 27 E_6 multiplet and form two copies of a $5+\overline{5}$ of the SU(5) subgroup of E_6, and one colour triplet Higgs particle. Due to Renormalization Group effects, the extra states increase the value of the Yukawa coupling constant for $S_3h_3h_3$ at low energies, and hence increase the mass of the lightest CP even Higgs boson [7].

Since S_3 is assumed to get a VEV at the TeV scale, this suggests that the $\mathcal{D}_{1,2,3}$ particles from the three copies of the 27 multiplet should transform as Δ_{27} singlets, so they may all acquire TeV scale masses. If instead we assumed them to be Δ_{27} triplets then at least one of their masses would be expected to be lower than the electroweak symmetry breaking scale, in violation of the direct experimental limits. This is because we would expect the effective couplings $S_3\mathcal{D}_{1,2,3}\mathcal{D}_{1,2,3}$, with S_3 obtaining a VEV at the TeV scale, to have a strongly hierarchical mass structure, as in the case of ordinary quarks, with at least the first generation, D_1, possibly having a mass lower the electroweak breaking scale. Instead, with $\mathcal{D}_{1,2,3}$ as Δ_{27} singlets, they will all obtain TeV scale masses from the (unsuppressed) superpotential terms $S_3\mathcal{D}_{1,2,3}\mathcal{D}_{1,2,3}$. Similarly, we take the first two generations of h from the fundamental 27 multiplets, which we denote by $h_{1,2}$, to transform as Δ_{27} singlets so that they obtain TeV scale masses from the $S_3h_{1,2}h_{1,2}$ superpotential terms.[3]

2.4 Proton decay and colour triplet Higgs decay

Here we show how the family symmetry can help to suppress proton decay arising from the light colour triplet exchange. The Pati-Salam $\mathcal{D}_{1,2,3}$ particles, which we shall refer to as colour triplet Higgs, decompose to $D_{1,2,3} \equiv (3,1)_{-\frac{1}{3}}$ and $\overline{D}_{1,2,3} \equiv (\overline{3},1)_{\frac{1}{3}}$ multiplets of the Standard Model and will cause proton decay unless the effective interactions $D_{1,2,3}QQ + \overline{D}_{1,2,3}u^cd^c$ or $\overline{D}_{1,2,3}QL + D_{1,2,3}\nu^cd^c + D_{1,2,3}e^cu^c$, which are allowed by the E_6 superpotential 27^3, are heavily suppressed or forbidden [7, 8]. These operators are always present in GUTs and SUSY GUTs, see Raby in [24]. However, in the exact Δ_{27} symmetry limit, operators of the form $\mathcal{D}FF$ and $\mathcal{D}F^cF^c$ are forbidden, since F, F^c are family triplets while $\mathcal{D}$ are family singlets.

[3]Note that the first two generations of h and $\mathcal{D}$ can fit inside a 10_{-1} multiplet of SO(10) $\times$ U(1)$_\psi$, but the third generations cannot due to opposite Z_2^H parity assignments. Also note that the required TeV scale VEV of S_3 implies an effective μ-term of similar magnitude, leading to a slight tuning required for electroweak symmetry breaking.

Once the Δ_{27} family symmetry is broken however, proton decay operators will reappear suppressed by flavon and other VEVs, and it becomes a quantitative question whether these operators are sufficiently suppressed. With the Z_2^H and Δ_{27} symmetries chosen as in table 1, the only way to generate these proton-decay inducing terms is from higher order terms involving flavons (to repair the Δ_{27} symmetry), and the E_6 singlet Σ (to repair the Z_2^H symmetry). Taking Σ to have $\mathrm{U}(1) = +5$ and $Z_2 = -1$, the smallest suppressed proton decay terms are:[4]

$$\frac{1}{M_S M_d^6}\Sigma\mathcal{D}_{1,2,3}F^iF^j\overline{\phi}_{123i}\overline{\phi}_{23j}(\phi_{123}^k\overline{\phi}_{3k})(\phi_1^l\overline{\phi}_{3l}) \;+\; (F^{i,j}\to F^{ci,j}) \tag{2.9}$$

$$\frac{1}{M_S M_d^6}\Sigma\mathcal{D}_{1,2,3}(\epsilon_{ijk}F^{ci}\phi_{123}^j\phi_3^k)(\epsilon_{lmn}F^{cl}\phi_1^m\phi_3^n)(\phi_1^l\overline{\phi}_{123l}) \;+\; (F^{i,j}\to F^{ci,j}) \tag{2.10}$$

These operators are suppressed by the square of a string scale M_S, which we take to be of order $10^{17.5}\,\mathrm{GeV}$. We assume that this type of suppression can be achieved due to the fact that the messengers that couple the Σ particle to the $F^cF^cD_{1,2,3}$ superpotential term are different to the messengers that couple the flavons and H_R to the quarks and leptons in the Yukawa and Majorana interactions of sections 2.1 and 2.2. We assume that the former messengers reside at the unification scale which we take to be a string scale $M_S \sim 10^{17.5}\,\mathrm{GeV}$, see section 3.2 for further discussion. The effective terms $F^cF^cD_{1,2,3}$ are then suppressed by a factor of about $\epsilon_d^6\epsilon_3^2\frac{<\Sigma>}{M_S}$, which, for $\epsilon_d \sim 0.13$, $\epsilon_3 \sim 0.8$, $<\Sigma>\sim 10^{11}\,\mathrm{GeV}$, and $M_S \sim 10^{17.5}\,\mathrm{GeV}$, is around 10^{-12}. This level of suppression should be just sufficient to prevent proton decay from being observable in present experiments if the colour triplets have mass greater than about $1.5\,\mathrm{TeV}$ [8].[5] We emphasize that the 10^{-12} level of suppression is only a rough order of magnitude calculation and can be determined from a number of sources, for example, the $d = 6$ proton decay operators in R-parity violating models [22], and the $d = 6$ proton decay operators in Grand Unified Theories with doublet-triplet splitting [23]. The present experimental limit on the $d = 6$ proton decay operator $p \to \pi^0 e^+$ is 5.0×10^{33} yrs [24].

To prevent the colour triplets from decaying with a lifetime smaller than 0.1 s the interactions $FF\mathcal{D}_{1,2,3} + F^cF^c\mathcal{D}_{1,2,3}$ should be suppressed by no more than roughly 10^{-12} or 10^{-13} (using order of magnitude calculations from [8]). A lifetime longer than about 0.1s for the colour triplets could cause problems for nucleosynthesis. The amount of Yukawa suppression for these interactions is thus uniquely set to be about 10^{-12} with the upper limit set by the proton decay and the lower limit set by colour triplet decay requirements. This small allowed window of couplings warrants a more detailed analysis of both proton

[4]Replacing $D_{1,2,3}$ with $h_{1,2}$, and F^iF^j by F^iF^{cj}, in eq. (2.9)–(2.10) gives the least suppressed FCNCs that are induced by the 'non-Higgses' $h_{1,2}$ [7].

[5]In [8] it was calculated that the level of suppression required to prevent proton decay was roughly 10^{-8} rather than 10^{-12}. The suppression of 10^{-8} used in [8] only prevents proton decay if the grand unified coupling constant for the interactions between the colour triplets and the up and down quarks was of the same order of magnitude as the up and down Yukawa coupling constant in the Higgs sector. This is not possible in the $\mathrm{ME_6SSM}$ with family symmetry model however since the up and down Yukawa coupling constants are generated by the flavon structure. We therefore require a suppression of $\sim |Y_{u,d}| \times 10^{-8} \sim 10^{-12}$ for the appropriate interactions.

decay and triplet decay, which we hope will be performed in the future, since it will lead to testable predictions for proton decay. The long lived TeV scale colour triplet states, which will be quasi-stable at colliders, lead to striking signatures at the LHC [25].

The above solution to triplet-Higgs-induced proton decay is very different from the solution used in conventional SUSY GUTs. Generically the solution is to make $\mathcal{D}_{1,2,3}$ very heavy (usually above the GUT scale) using doublet-triplet splitting. However, no such doublet-triplet splitting is allowed in this theory since gauge anomalies for the low energy $\mathrm{U}(1)_X$ gauge group would be created [8], and instead the proton decay is suppressed by the symmetries of the model (in particular the Δ_{27} family symmetry).

2.5 R-parity and $H_R + \overline{H}_R$ Mass

Not all the components of H_R and $\overline{H}_R$ obtain mass by absorbing the broken Pati-Salam gauge bosons when they acquire vacuum expectation values in the right-handed neutrino direction. To give the rest of H_R and $\overline{H}_R$ (and H_L and $\overline{H}_L$ from the SO(10) multiplets 16_H and $\overline{16}_H$) mass, we have included a singlet M in table 1. This singlet is assumed to get a GUT scale VEV, giving mass to $16_H + \overline{16}_H$ from the superpotential term $M 16_H \overline{16}_H$. Since M carries a $\mathrm{U}(1)_R$ charge of +2, its VEV breaks $\mathrm{U}(1)_R$ to an R-parity. This R-parity is the same as that in the ME$_6$SSM, which is a generalization of R-parity in the MSSM. This R-parity keeps the LSP stable, thus providing a dark matter candidate.

2.6 h', $\overline{h}'$ Mass in the E$_6$SSM

In the E$_6$SSM, to prevent the two additional electroweak doublets h' and $\overline{h}'$ from introducing gauge anomalies for the $\mathrm{U}(1)_N$ gauge group, they are assumed have opposite $\mathrm{U}(1)_N$ charges. These particles effectively reintroduce a μ'-problem since there is no simple mechanism that explains why these particles have low energy masses. Here we give the particles mass by assuming that the E_6 singlet Σ couples to the h' and $\overline{h}'$ through the non-renormalizable term $(1/M_S)\Sigma\Sigma h'\overline{h}'$ and obtains a vacuum expectation value at 10^{11} GeV. This gives h' and $\overline{h}'$ the correct scale of mass for gauge coupling unification to occur at the GUT scale (see the third reference in [7]).

The way in which h' and $\overline{h}'$ transform under the Pati-Salam, $\mathrm{U}(1)_\psi$, and other symmetries is presented in table 1, which contains the total particle spectrum of the E$_6$SSM (and ME$_6$SSM) with Δ_{27} family symmetry.[6]

3. Gauge coupling unification

3.1 Unification and symmetry breaking in the E$_6$SSM

In this subsection we briefly discuss the pattern of symmetry breaking for the E$_6$SSM with a Δ_{27} family symmetry model. Adding the extra electroweak states h' and $\overline{h}'$ at the TeV scale to the three copies of a 27 causes the Standard Model gauge coupling constants to unify at the conventional GUT scale but with a higher value than the MSSM prediction

[6] In table 1 h' and $\overline{h}'$ are chosen to transform as $(1,2,1)_x$ and $(1,2,1)_{-x}$ Pati-Salam representations respectively where x is a real number. Such multiplets cannot be derived from E_6 multiplets.

for the unification gauge coupling constant (see the third reference of [7]). This of course requires that the $SU(2)_R$ and $SU(4)_{PS}$ subgroups of the E_6 symmetry be broken at the same scale (the GUT scale). However, as discussed further in the following section (3.2), we expect the $SU(2)_R$ and $SU(4)_{PS}$ groups to be broken by separate mechanisms so that the $SU(2)_L$ messengers and up and down $SU(2)_R$ messengers have different masses but the quark and lepton components of the $SU(4)_{PS}$ messengers have the same masses, as required for section 2.1. To achieve this we assume that, at the GUT scale, the VEV of the $H_R + \overline{H}_R$ multiplets breaks $SU(4)_{PS}$ to $SU(3)_c \times U(1)_{B-L}$, and the $SU(2)_R$ is broken to $U(1)_{\tau_R^3}$ by a Wilson-line [12].[7] This will give the up $SU(2)_R$ messengers masses smaller than the GUT scale. Therefore, to compensate for the effect on the running of the Standard Model gauge coupling constants caused by the up $SU(2)_R$ messengers (which would upset unification), we would require additional messengers below the GUT scale that, together with the up $SU(2)_R$ messengers, form a *complete* 10 multiplet of SU(5). The messengers below the GUT scale would increase the MSSM prediction for the value of the unification gauge coupling constant but keep the unification scale as the conventional GUT scale. Of course too many messengers, and too small messenger masses, would cause the Standard Model gauge coupling constants to blow up before they unify. Here we simply assume that the minimal number of messengers required to generate the correct quark and lepton masses and mixing angles does not prevent the unification of the Standard Model gauge coupling constants at the GUT scale.

3.2 Unification and symmetry breaking in the ME$_6$SSM

In this subsection we discuss the pattern of symmetry breaking for the ME$_6$SSM with a Δ_{27} family symmetry model and, using two simple toy models, demonstrate that gauge coupling unification at the string scale could be possible. In the ME$_6$SSM the E_6 symmetry is assumed to be broken at the Planck scale to a left-right symmetric Pati-Salam gauge group $SU(4)_{PS} \times SU(2)_L \times SU(2)_R \times D_{LR}$ (a maximal subgroup of SO(10)) and an Abelian gauge group $U(1)_\psi$. The left-right symmetric gauge group is then broken to the Standard Model gauge group with an additional Abelian gauge group $U(1)_X$, which is a combination of the charge of the $U(1)_\psi$ group, the diagonal generator τ_R^3 of the $SU(2)_R$ group, and the diagonal generator associated with the $U(1)_{B-L}$ subgroup of $SU(4)_{PS}$ defined by $SU(4)_{PS} \to SU(3)_c \times U(1)_{B-L}$. This breaking is achieved by the ME$_6$SSM equivalent to the $H_R + \overline{H}_R$ particles from gaining VEVs in the right-handed neutrino directions. At the scale of this symmetry breaking the gauge couplings of the Abelian groups $U(1)_{B-L}$, $U(1)_{\tau_R^3}$ and $U(1)_Y$ must satisfy the following equation [8]:

$$\frac{5}{\alpha_Y} = \frac{3}{\alpha_{\tau_R^3}} + \frac{2}{\alpha_{B-L}} \tag{3.1}$$

[7]In addition to breaking the $SU(4)_{PS}$ symmetry, the VEV of the $H_R + \overline{H}_R$ multiplets will also mix the $U(1)_{B_L}$, $U(1)_{\tau_R^3}$ and $U(1)_\psi$ groups to create $U(1)_Y$ and the $U(1)_N$ group of the E$_6$SSM. The $U(1)_N$ group of the E$_6$SSM is generated, rather than the $U(1)_X$ group of the ME$_6$SSM, because the gauge coupling constants of the Pati-Salam (and $U(1)_\psi$) symmetries are equal at the symmetry breaking scale [8].

For the ME_6SSM this is equivalent to [8]:

$$\frac{5}{\alpha_Y} = \frac{3}{\alpha_{2R}} + \frac{2}{\alpha_{4PS}} \tag{3.2}$$

Using $\alpha_{4PS} = \alpha_3$ and $\alpha_{2L} = \alpha_{2R}$ from the left-right symmetry, the above equation can be written solely in terms of Standard Model gauge coupling constants. The scale of the Pati-Salam symmetry is therefore determined by running the Standard Model gauge couplings up until they satisfy this boundary equation. With three copies of a 27 multiplet at low energies this scale is found to be $10^{16.4}$ GeV at the two-loop order [8].

When we include the Δ_{27} family symmetry to the ME_6SSM, the pattern of symmetry breaking is likely to change from the above discussion. This is because the $SU(2)_R$ and $SU(4)_{PS}$ groups must be broken by separate mechanisms so that the $SU(2)_L$ messengers and up and down $SU(2)_R$ messengers have different masses but the quark and lepton components of the $SU(4)_{PS}$ messengers have the same masses, as required for section 2.1. To achieve this we assume that the VEV of the $H_R + \overline{H}_R$ multiplets breaks $SU(4)_{PS}$ to $SU(3)_c \times U(1)_{B-L}$, and that $SU(2)_R$ is broken to $U(1)_{\tau_R^3}$ by a Wilson-line at some compactification scale [12].[8] To prevent the messengers from altering the running of the gauge couplings of the ME_6SSM to a large degree, we expect that M_d should be of order or greater than the $SU(4)_{PS}$ breaking scale. It then follows that, to generate $M_u \sim 3M_d$, the compactification scale at which $SU(2)_R$ is broken to $U(1)_{\tau_R^3}$ should be around three times greater than M_d.

The pattern of symmetry breaking in this case is thus expected to proceed as follows: the $SU(2)_R$ group is broken to $U(1)_{\tau_R^3}$ at a compactification scale M_C, which, along with the $SU(4)_{PS} \times SU(2)_L \times U(1)_\psi$ symmetry, is broken at a lower scale to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X$ by the $H_R + \overline{H}_R$ particles. We also expect the left-right discrete symmetry to be broken since the left-handed messengers are heavier than and right-handed messengers. In realistic models of the ME_6SSM with Δ_{27} family symmetry, we therefore do not expect the scale of G_{4221} symmetry breaking to be determined uniquely using eq. (3.1) since there is no longer a symmetry that sets $\alpha_{\tau_R^3}$ equal to α_{2L} at this scale.

The $H_R + \overline{H}_R$ particles also transform under the Δ_{27} family symmetry and get VEVs in the third component so that they break the Δ_{27} symmetry at the same scale as the $G_{4211} \equiv SU(4)_{PS} \times SU(2)_L \times U(1)_{\tau_R^3} \times U(1)_\psi$ symmetry. The remaining part of the family symmetry, which is a subgroup of Δ_{27}, will be broken by the VEV of the $\phi_{23} + \overline{\phi}_{23}$ flavons at the scale $\epsilon_d M_d$ where the right-handed messengers mass M_d should be above the Δ_{27} symmetry breaking scale otherwise wavefunction insertions of the invariant operator $\phi_3 \phi_3^\dagger / M_R^2$ on a third family propagator can spoil the perturbative expansion if $< \phi_3 > \; > M_R$ [12].

The scale of the E_6 symmetry breaking in the ME_6SSM is also expected to be modified when the Δ_{27} symmetry is included. Instead of Planck scale E_6 symmetry breaking, we expect the E_6 symmetry to be broken at a string scale. This is mainly due to the number of

[8] One could alternatively consider the VEV of H_{45} to break $SU(4)_{PS}$ to $SU(3)_c \times U(1)_{B-L}$. This depends on whether the VEV of H_{45} is chosen to be at a greater or smaller energy scale than the $H_R + \overline{H}_R$ VEV. In [11] and (the second reference in) [12], for example, the H_{45} VEV is taken to be of order $3M_d$ and $3\epsilon_d M_d$ respectively.

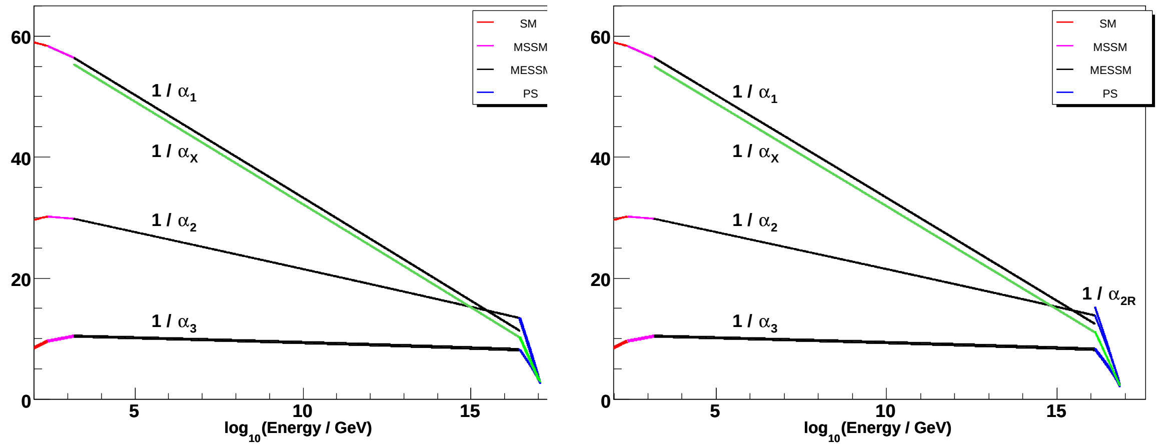

Figure 1: This figure illustrates the two-loop RGEs running of the gauge coupling constants for two models based on the ME$_6$SSM with Δ_{27} family symmetry. It demonstrates that unification can be possible using a basic QFT description although we expect additional effects such as extra dimensions to change the running of the gauge couplings at higher energies and therefore to change the scale of unification. The two models are described in detail in section 3.2. The left panel is for a model with left-right symmetric intermediate G_{4221} symmetry, whereas the right panel is for a more realistic non-symmetric G_{4221} symmetry. The scales of unification and G_{4221} symmetry breaking are of order $10^{17.1}$, $10^{16.9}$ GeV and $10^{16.4}$, $10^{16.1}$ GeV for the left, right panel respectively.

additional particles (messengers) to the ME$_6$SSM states at and above the G_{4211} symmetry scale, which are required for the Δ_{27} family symmetry to accurately describe the observed quark and fermion masses and mixing angles. These extra states cause the gauge coupling constants to increase rapidly above the G_{4211} symmetry breaking scale, bringing forward the unification scale. Other modifications to the E_6 symmetry breaking scale in the ME$_6$SSM will come from extra dimensions above the compactification scale, the running of the gauge coupling constant for the Abelian $\mathrm{U}(1)_{\tau^3_R}$ group, and the breaking of the left-right discrete symmetry at the compactification scale.

Unification of the gauge coupling constants may in fact no longer be possible when all of these changes from the ME$_6$SSM are calculated, but in figure 1 we demonstrate that gauge coupling unification still occurs for two simple toy models of the ME$_6$SSM with Δ_{27} symmetry. We make the approximation that the compactification scale is equal to the G_{4211} symmetry breaking scale. Both toy models therefore have an intermediate G_{4221} symmetry as in the ME$_6$SSM. However, for the toy model in the right panel of figure 1, we assume that the left-right discrete symmetry is broken at the unification scale due to the different masses for the left-handed and right-handed messengers. Furthermore, we also neglect any effects from extra dimensions above the compactification scale. In both panels of figure 1 we assume that three copies of an E_6 27 multiplet, which contain all the MSSM states as well as new (non-MSSM) states, have mass at low energies and, following the ME$_6$SSM, we take effective MSSM and non-MSSM thresholds of 250 GeV and 1.5 TeV respectively.

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At the $\Delta_{27} \times G_{4221}$ symmetry breaking scale, we also assume additional particles that break the symmetry and play a part in the Δ_{27} family symmetry's description of quark and lepton masses. In the left panel we take these extra particles to consist of all the G_{4221} states from five copies of $27 + \overline{27}$ multiplets, except for the $(6,1,1)_{\frac{1}{2}} + (6,1,1)_{-\frac{1}{2}}$ states which we assume have mass at the unification scale, as well as all the flavons given in table 1 and a left-handed partner for $\overline{\phi}_3$. The additional $27 + \overline{27}$ states contain the $16_H + \overline{16}_H$ particles that break the $\Delta_{27} \times G_{4221}$ symmetry and provide the Majorana interactions, the $16 + \overline{16}$ particles that give the H_{45} as a composite, and messengers that also transform as a $16 + \overline{16}$ of SO(10). We assume that H_{45} is a composite of a $16 + \overline{16}$ state since a fundamental H_{45} particle (and its left-handed partner) would affect the running of the $\mathrm{SU}(4)_{\mathrm{PS}}$ gauge couplings by an amount that causes it to blow up before any unification of gauge couplings is possible, unless a large number of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ extra states are added to compensate for this. We would also need to explain why the rest of the 650 E_6 multiplet, that contains the H_{45}, have larger mass. On top of the five copies of the $27 + \overline{27}$ multiplets we also add additional Higgs messengers that transform as a triplet and an anti-triplet of the Δ_{27} family symmetry. These are required for unification of the gauge coupling constants.

For the right panel we include the same states as the left panel but without the left-handed messengers as these are expected to get much larger masses than their right-handed components. The scales of unification and G_{4221} symmetry breaking are at $10^{17.1}$, $10^{16.9}$ GeV and $10^{16.4}$, $10^{16.1}$ GeV for the left, right panel respectively. Note that the G_{4221} symmetry breaking scales are close to the Grand Unification scale in conventional GUTs, we thus denote the scale by M_{GUT}.

We emphasize that these toy models do not represent accurate predictions for the running of the gauge coupling constants of the $\mathrm{ME_6SSM}$ with Δ_{27} family symmetry and are only used to demonstrate that, with the inclusion of the Δ_{27} messenger states to the $\mathrm{ME_6SSM}$, gauge coupling unification is still possible but at a scale that is closer to the String scale than the Planck scale.

4. Summary

In this paper we have discussed models based on broken E_6 GUT with a Δ_{27} (a discrete subgroup of SU(3)) family symmetry broken close to the GUT scale. To provide realistic models we also require additional symmetries, including an R-symmetry which results in a conserved R-parity. The models combine the $\mathrm{ME_6SSM}$ and $\mathrm{E_6SSM}$ proposed in [7, 8] with the Δ_{27} family symmetry approach of [14]. The resulting synthesis is very powerful and predictive, and solves a number of problems facing the MSSM, including the little fine-tuning problem, the μ-problem and the flavour problem. The solution to the μ-problem requires an additional low energy $\mathrm{U}(1)_X$ gauge group, under which right-handed neutrinos are neutral, allowing a conventional see-saw mechanism. The Δ_{27} accounts for the quark and lepton masses and mixing angles, with tri-bimaximal neutrino mixing resulting from vacuum alignment and constrained sequential dominance. Note that we have considered both the $\mathrm{ME_6SSM}$ and $\mathrm{E_6SSM}$ formulated in terms of a Pati-Salam symmetry (and an Abelian gauge group $\mathrm{U}(1)_\psi$) yielding the Standard Model gauge group (and an Abelian

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gauge group $\mathrm{U}(1)_X$) below the conventional GUT scale. In the case of the E_6SSM the gauge group $\mathrm{U}(1)_X$ is identical to $\mathrm{U}(1)_N$ of [7] since the gauge couplings are unified at M_{GUT}.

The main phenomenological difference between the E_6SSM discussed here (broken via the Pati-Salam chain) and the E_6SSM discussed in [7] (broken via the SU(5) chain) arises from the physics of the colour triplet Higgs couplings. In the original E_6SSM [7], exact Z_2^L or Z_2^B symmetries are allowed corresponding to the colour triplet states coupling as diquarks or leptoquarks, effectively preventing proton decay, while allowing rapid colour triplet decay. However, the Pati-Salam symmetry assumed here for both the ME_6SSM and E_6SSM, prevents the use of the Z_2^B or Z_2^L. Instead, in both the ME_6SSM and E_6SSM, the colour triplet Yukawa couplings must be suppressed down to the level of 10^{-12}, as required to sufficiently suppress proton decay whilst allowing the states to decay before nucleosynthesis. This is achieved by the symmetries of the model, with the Δ_{27} family symmetry playing an important role in helping to achieve the required degree of suppression. The highly suppressed couplings imply long lived TeV mass colour triplets, with a lifetime typically about 0.1 sec for example, providing a striking signature of these models at the LHC.

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References

[1] D.J.H. Chung et al., *The soft supersymmetry-breaking lagrangian: theory and applications*, *Phys. Rept.* **407** (2005) 1 [hep-ph/0312378].

[2] J.R. Ellis, S. Kelley and D.V. Nanopoulos, *Probing the desert using gauge coupling unification*, *Phys. Lett.* **B 260** (1991) 131;
P. Langacker and M.-X. Luo, *Implications of precision electroweak experiments for $M(t)$, ρ_0, $\sin^2(\Theta_W)$ and grand unification*, *Phys. Rev.* **D 44** (1991) 817;
U. Amaldi, W. de Boer and H. Furstenau, *Comparison of grand unified theories with electroweak and strong coupling constants measured at LEP*, *Phys. Lett.* **B 260** (1991) 447;
F. Anselmo, L. Cifarelli, A. Peterman and A. Zichichi, *The effective experimental constraints on M (SUSY) and M (GUT)*, *Nuovo Cim.* **A104** (1991) 1817; *The evolution of gaugino masses and the SUSY threshold*, *Nuovo Cim.* **A105** (1992) 581.

[3] S.P. Martin, *A supersymmetry primer*, hep-ph/9709356.

[4] J.R. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, *Higgs bosons in a nonminimal supersymmetric model*, *Phys. Rev.* **D 39** (1989) 844;
D. Suematsu and Y. Yamagishi, *Radiative symmetry breaking in a supersymmetric model with an extra* U(1), *Int. J. Mod. Phys.* **A 10** (1995) 4521 [hep-ph/9411239];
M. Cvetič and P. Langacker, *Implications of abelian extended gauge structures from string models*, *Phys. Rev.* **D 54** (1996) 3570 [hep-ph/9511378]; *New gauge bosons from string models*, *Mod. Phys. Lett.* **A 11** (1996) 1247 [hep-ph/9602424];
D. Suematsu, *Vacuum structure of the μ-problem solvable extra* U(1) *models*, *Phys. Rev.* **D 59** (1999) 055017 [hep-ph/9808409].

[5] J.E. Kim and H.P. Nilles, *The μ problem and the strong CP problem*, *Phys. Lett.* **B 138** (1984) 150;
E.J. Chun, J.E. Kim and H.P. Nilles, *A Natural solution of the μ problem with a composite axion in the hidden sector*, *Nucl. Phys.* **B 370** (1992) 105.

[6] See e.g., A. Djouadi et al., *Benchmark scenarios for the NMSSM*, `arXiv:0801.4321`.

[7] S.F. King, S. Moretti and R. Nevzorov, *Theory and phenomenology of an exceptional supersymmetric standard model*, *Phys. Rev.* **D 73** (2006) 035009 [`hep-ph/0510419`]; *Exceptional supersymmetric standard model*, *Phys. Lett.* **B 634** (2006) 278 [`hep-ph/0511256`];
S.F. King, S. Moretti and R. Nevzorov, *Gauge coupling unification in the exceptional supersymmetric standard model*, *Phys. Lett.* **B 650** (2007) 57 [`hep-ph/0701064`].

[8] R. Howl and S.F. King, *Planck scale unification in a supersymmetric standard model*, *Phys. Lett.* **B 652** (2007) 331 [`arXiv:0705.0301`]; *Minimal E_6 supersymmetric standard model*, *JHEP* **01** (2008) 030 [`arXiv:0708.1451`].

[9] For recent reviews see e.g., S.F. King, *Neutrino physics*, `arXiv:0712.1750`;
R.N. Mohapatra and A.Y. Smirnov, *Neutrino mass and new physics*, *Ann. Rev. Nucl. Part. Sci.* **56** (2006) 569 [`hep-ph/0603118`];
R.N. Mohapatra et al., *Theory of neutrinos: a white paper*, *Rept. Prog. Phys.* **70** (2007) 1757 [`hep-ph/0510213`];
S.F. King, *Neutrino mass models*, *Rept. Prog. Phys.* **67** (2004) 107 [`hep-ph/0310204`];
G. Altarelli and F. Feruglio, *Models of neutrino masses and mixings*, *New J. Phys.* **6** (2004) 106 [`hep-ph/0405048`];
R.N. Mohapatra, *Understanding neutrino masses and mixings within the seesaw framework*, `hep-ph/0306016`.

[10] See for example, R. Barbieri, L.J. Hall, S. Raby and A. Romanino, *Unified theories with* U(2) *flavor symmetry*, *Nucl. Phys.* **B 493** (1997) 3 [`hep-ph/9610449`];
M.-C. Chen and K.T. Mahanthappa, *From CKM matrix to MNS matrix: a model based on supersymmetric* $\mathrm{SO}(10) \times \mathrm{U}(2)_F$ *symmetry*, *Phys. Rev.* **D 62** (2000) 113007 [`hep-ph/0005292`];
G. Altarelli, F. Feruglio and I. Masina, *From minimal to realistic supersymmetric* SU(5) *grand unification*, *JHEP* **11** (2000) 040 [`hep-ph/0007254`];
J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, *Minimal mixing of quarks and leptons in the* SU(3) *theory of flavour*, *Nucl. Phys.* **B 626** (2002) 307 [`hep-ph/0109156`];
J. Kubo, A. Mondragon, M. Mondragon and E. Rodriguez-Jauregui, *The flavor symmetry*, *Prog. Theor. Phys.* **109** (2003) 795 [*Erratum ibid.* **114** (2005) 287] [`hep-ph/0302196`];
G.L. Kane, S.F. King, I.N.R. Peddie and L. Velasco-Sevilla, *Study of theory and phenomenology of some classes of family symmetry and unification models*, *JHEP* **08** (2005) 083 [`hep-ph/0504038`];
R. Dermisek and S. Raby, *Bi-large neutrino mixing and CP-violation in an* SO(10) *SUSY GUT for fermion masses*, *Phys. Lett.* **B 622** (2005) 327 [`hep-ph/0507045`];
R. Dermisek, M. Harada and S. Raby, SO(10) *SUSY GUT for fermion masses: lepton flavor and CP-violation*, *Phys. Rev.* **D 74** (2006) 035011 [`hep-ph/0606055`].

[11] I. de Medeiros Varzielas and G.G. Ross, SU(3) *family symmetry and neutrino bi-tri-maximal mixing*, *Nucl. Phys.* **B 733** (2006) 31 [`hep-ph/0507176`].

[12] S.F. King and G.G. Ross, *Fermion masses and mixing angles from* SU(3) *family symmetry*, *Phys. Lett.* **B 520** (2001) 243 [`hep-ph/0108112`]; *Fermion masses and mixing angles from* SU(3) *family symmetry and unification*, *Phys. Lett.* **B 574** (2003) 239 [`hep-ph/0307190`].

[13] S.F. King, *Predicting neutrino parameters from* SO(3) *family symmetry and quark-lepton unification*, *JHEP* **08** (2005) 105 [hep-ph/0506297];
S.F. King and M. Malinsky, *Towards a complete theory of fermion masses and mixings with* SO(3) *family symmetry and* 5*D* SO(10) *unification*, *JHEP* **11** (2006) 071 [hep-ph/0608021].

[14] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Neutrino tri-bi-maximal mixing from a non-abelian discrete family symmetry*, *Phys. Lett.* **B 648** (2007) 201 [hep-ph/0607045].

[15] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Tri-bimaximal neutrino mixing from discrete subgroups of* SU(3) *and* SO(3) *family symmetry*, *Phys. Lett.* **B 644** (2007) 153 [hep-ph/0512313];
S.F. King and M. Malinsky, *A_4 family symmetry and quark-lepton unification*, *Phys. Lett.* **B 645** (2007) 351 [hep-ph/0610250];
G. Altarelli, F. Feruglio and C. Hagedorn, *A SUSY* SU(5) *grand unified model of tri-bimaximal mixing from A_4*, *JHEP* **03** (2008) 052 [arXiv:0802.0090].

[16] S.F. King, *Atmospheric and solar neutrinos with a heavy singlet*, *Phys. Lett.* **B 439** (1998) 350 [hep-ph/9806440]; *Large mixing angle MSW and atmospheric neutrinos from single right-handed neutrino dominance and U(1) family symmetry*, *Nucl. Phys.* **B 576** (2000) 85 [hep-ph/9912492]; *Constructing the large mixing angle MNS matrix in see-saw models with right-handed neutrino dominance*, *JHEP* **09** (2002) 011 [hep-ph/0204360]; *Predicting neutrino parameters from* SO(3) *family symmetry and quark-lepton unification*, *JHEP* **08** (2005) 105 [hep-ph/0506297]; *Invariant see-saw models and sequential dominance*, *Nucl. Phys.* **B 786** (2007) 52 [hep-ph/0610239].

[17] G.G. Ross and O. Vives, *Yukawa structure, flavour and CP-violation in supergravity*, *Phys. Rev.* **D 67** (2003) 095013 [hep-ph/0211279];
G.G. Ross, L. Velasco-Sevilla and O. Vives, *Spontaneous CP-violation and non-Abelian family symmetry in SUSY*, *Nucl. Phys.* **B 692** (2004) 50 [hep-ph/0401064];
S. Antusch, S.F. King and M. Malinsky, *Solving the SUSY flavour and CP problems with* SU(3) *family symmetry*, arXiv:0708.1282.

[18] H. Georgi and C. Jarlskog, *A new lepton-quark mass relation in a unified theory*, *Phys. Lett.* **B 86** (1979) 297.

[19] G.G. Ross and L. Velasco-Sevilla, *Symmetries and fermion masses*, *Nucl. Phys.* **B 653** (2003) 3 [hep-ph/0208218].

[20] G. Ross and M. Serna, *Unification and fermion mass structure*, arXiv:0704.1248.

[21] F. Plentinger and W. Rodejohann, *Deviations from tribimaximal neutrino mixing*, *Phys. Lett.* **B 625** (2005) 264 [hep-ph/0507143].

[22] R. Barbier et al., *R-parity violating supersymmetry*, *Phys. Rept.* **420** (2005) 1 [hep-ph/0406039].

[23] P. Nath and P. Fileviez Perez, *Proton stability in grand unified theories, in strings and in branes*, *Phys. Rept.* **441** (2007) 191 [hep-ph/0601023].

[24] Particle Data Group collaboration, W.M. Yao et al., *Review of particle physics*, *J. Phys.* **G 33** (2006) 1.

[25] A. Nisati, S. Petrarca and G. Salvini, *On the possible detection of massive stable exotic particles at the LHC*, *Mod. Phys. Lett.* **A 12** (1997) 2213 [hep-ph/9707376].